# Laser Illuminated Feature Position Determination and Calibration Approach for PIXL

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***Abstract*—This paper describes several approaches for the calibration of translational and rotational alignment errors between a camera and a set of laser illuminators for a position determination system. Detailed algorithms are derived and covariance of calibration residuals in both error state and target position determination accuracy are obtained. An example with system design parameters is given to demonstrate the effectiveness of the approaches. The algorithm can be used for any terrestrial or space position determination instruments employing a camera/laser triangulation system and is proposed, as one application, for the PIXL instrument of JPL's MARS 2020 Rover project.**

***Keywords—error calibration; structured light system; Mars exploration; position determination; alignment error; error estimation***

## I. Introduction

The Planetary Instrument for X-ray Lithochemistry (PIXL), as part of JPL's MARS 2020 Rover project, is a precision X-Ray Fluorescence instrument for measuring fine scale chemical variations in rocks and soils of the Martian surface [1,2]. Surface target position determination and registration are critical PIXL requirements and the approach to determine the surface target position is via a structured light system (SLS).

The PIXL SLS includes a camera and a large number of laser beams. With the lasers illuminating the surface and the camera capturing the laser images, triangulation algorithms can be used to determine the surface target location in the instrument frame. A general depiction of the PIXL SLS is provided in Figure 1:

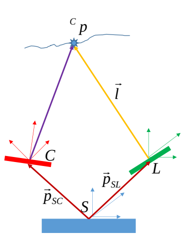


Figure 1 – Camera and Laser Configuration

During PIXL operation, multiple laser beams are projected on the target surface at different locations. The camera images of the laser spots can be used to determine the surface locations of the laser spots via triangulation. The detailed design of the camera/laser systems can be found in [3,4].

The research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.



Most existing literature on SLS calibration concentrated on using structured lights to estimate the intrinsic camera parameters (e.g. [5]). This paper, however, is focused on calibrating the co-alignment errors between the camera and laser system. The knowledge of the co-alignment between the camera frame and the laser frames and their alignment to the sensor head frame (and the x-ray) are critical in ensuring the PIXL pointing performance. Ground surveys are typically performed to measure the alignment errors. However, the alignment measurement residuals are always present and often calibrations are required to further improve the accuracy of the alignment error knowledge to improve the determination accuracy during mission phase. This paper describes an approach to systematically calibrate the alignment errors between the camera and laser coordinate frames for the purpose of improving the overall target position determination accuracy performance. Several algorithms are derived and their performances are compared. Numerical simulation results are also provided.

The proposed approach has several advantages over the existing methods. Firstly, the calibration method estimates a set of intrinsic alignment error variables existing between the camera and laser frames. The estimated error knowledge can be used to correct the PIXL target position determination error over the entire instrument pose space. Unlike a table look-up approach, this correction can be precise and does not suffer the quantization problem associated with a correction table. Secondly, the algorithms used for the estimation of the errors are robust and converge fast with limited number of measurements, and do not employ any nonlinear search algorithm. Consequently, they are simple enough to be embedded in flight software in case of a need for post-launch re-calibration with available calibration targets during the rover mission. Finally, it is also noted that although this approach is developed specifically for the Mars 2020 PIXL project, it can be used for alignment error co-calibration of any structured light system, and in principle, for any camera based triangulation position determination system in space or terrestrial applications.

The paper is organized as follows: In section 2, a pin-hole camera model is briefly described. Section 3 describes the general position determination algorithm without camera/laser alignment errors. Section 4 provides the alignment error modeling for calibration while alignment error estimation scheme is discussed in Section 5. Section 6 is devoted to the post-calibration alignment error and position determination accuracy analyses. Section 7 addresses the possibility of making use of the PIXL motion capabilities to provide the required calibration motion. A numerical example is shown in Section 8 to demonstrate the calibration performance. Finally concluding remarks and further development are provided in Section 9.

## II. A Pinhole Camera Model

For the purpose of the calibration, a simple pinhole camera model is used to model the camera measurements. The camera measurements are used for estimating the alignment errors between the camera and the individual laser beam. In general, the image of a laser surface spot location, as captured by the camera detector, can be described by its pixel horizontal and vertical locations in the camera's detector frame as

$$\begin{bmatrix} h_{pix} \\ v_{pix} \end{bmatrix} = \begin{bmatrix} -\dfrac{f}{s_h} & 0 & h_{pix0} \\ 0 & -\dfrac{f}{s_v} & v_{pix0} \end{bmatrix} \frac{1}{z} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \equiv M \frac{1}{z} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where $f$ is the focal length of the camera, $s_h$ and $s_v$ the scale factors in horizontal and vertical directions, and $h_{pix0}$ and $v_{pix0}$ the detector coordinates of the camera center.

## III. Position Estimation without Alignment Errors

### A. General Case

For the nominal case where there is no alignment errors between the camera and laser frames, the surface location of laser spot can be determined easily as follows. If the nominal camera frame and laser frame are both known relative to the sensor frame, the laser frame origin location expressed in camera frame is given by the following relationship:

$$^{c}p_{cl} = {^{c}p_{sl}} - {^{c}p_{sc}} = C_{cs}({^{s}p_{sl}} - {^{s}p_{sc}})$$

where $^{s}p_{sc}$ and $^{s}p_{sl}$ are the camera and laser frame origin locations in the sensor frame, expressed in the sensor frame, and $C_{cs}$ is the sensor frame to camera frame DCM. In the nominal case, this vector is assumed to be known precisely. The laser frame to laser spot vector, as expressed in the camera frame is then

$$^{c}l = {^{c}p} - {^{c}p_{cl}} = {^{c}p} - C_{cs}({^{s}p_{sl}} - {^{s}p_{sc}})$$

Define $l \equiv \left\| {^{c}l} \right\|$. The laser line-of-sight (LOS) unit vector in the camera frame is $^{c}u = C_{cs}C_{sl}{^{l}u}$. The laser spot location in the camera frame can be calculated as

$$^{c}p = {^{c}p_{cl}} + {^{c}l} = {^{c}p_{cl}} + l({^{c}u}) = {^{c}p_{cl}} + l(C_{cs}C_{sl}{^{l}u}) \equiv \bar{r} + l\bar{u}$$

where $\bar{r}$ is the known laser frame origin position vector, $\bar{u}$ the known laser LOS unit vector in the camera frame.

The position of the laser spot can be calculated using the camera measurements as follows. Combining the camera model and camera/laser frame geometric relationship, we have

$$\begin{bmatrix} h_{pix} \\ v_{pix} \end{bmatrix} = M\bar{r}\frac{1}{z} + M\bar{u}\frac{l}{z} = \underbrace{\begin{bmatrix} M\bar{u} & M\bar{r} \end{bmatrix}}_{V} \begin{bmatrix} l/z \\ 1/z \end{bmatrix} \equiv V \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

The position coordinates can be obtained as follows:

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = V^{-1} \begin{bmatrix} h_{pix} \\ v_{pix} \end{bmatrix} \text{ and } \begin{cases} z = 1/w_2 \\ \begin{bmatrix} x \\ y \end{bmatrix} = [M(:,1:2)]^{-1} z \begin{bmatrix} h_{pix} \\ v_{pix} \end{bmatrix} \end{cases}$$

### B. Special Case with Laser Alignment Information

In this section we provide the detailed formulae for target position calculation with the explicit knowledge of the laser frame. First, without loss of generality, we assume that the camera frame is aligned with the sensor frame and the laser frame can be defined using two Euler angles, (denoted as $a_l$ and $e_l$) rotations in a 2-1 sequence from the camera frame. The laser line-of-sight (LOS) unit vector, assumed to be along the laser frame z-axis, is given by

$$^{c}u = \begin{bmatrix} \cos e_l \sin a_l \\ -\sin e_l \\ \cos a_l \cos e_l \end{bmatrix}$$

If the laser frame origin is denoted as $^{c}p_{cl} = \begin{bmatrix} x_{cl} & y_{cl} & z_{cl} \end{bmatrix}^T$ and we assume $s_h = s_v \equiv s$. If we make use of the specific laser frame information, the camera measurement model is given by

$$c_m \equiv \begin{bmatrix} h_m \\ v_m \end{bmatrix} + \eta_m = -\frac{f}{s} \begin{bmatrix} \cos e_l \sin a_l & x_{cl} \\ -\sin e_l & y_{cl} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \eta_m$$

where $\eta_m$ is the camera measurement noise vector. The estimations of $w_1, w_2$ are given as

$$\begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} = -\frac{s}{f} \frac{\begin{bmatrix} h_m y_{cl} - v_m x_{cl} \\ h_m \sin e_l + v_m \cos e_l \sin a_l \end{bmatrix}}{x_{cl} \sin e_l + y_{cl} \cos e_l \sin a_l}$$

and the z-coordinate of the target is

$$\hat{z} = \frac{1}{\hat{w}_2} = -\left(\frac{f}{s}\right)\left(\frac{x_{cl} \sin e_l + y_{cl} \cos e_l \sin a_l}{h_m \sin e_l + v_m \cos e_l \sin a_l}\right)$$

### C. Position Determination Error Analysis

The laser spot location determination accuracy can be affected by many error sources. In this section, the effect of the camera measurement errors on the position determination is evaluated. Recall the camera measurement equation with measurement noise shown in the last section. The covariance of the error in the estimated $w_1, w_2$ variables is

$$\operatorname{cov}\begin{bmatrix} \Delta w_1 \\ \Delta w_2 \end{bmatrix} = \operatorname{cov}\left( \underbrace{-\frac{s}{f} \begin{bmatrix} \cos e_l \sin a_l & x_{cl} \\ -\sin e_l & y_{cl} \end{bmatrix}^{-1}}_{E_1} \eta_m \right) = E_1 B \operatorname{cov}[\eta_m] E_1^T$$

The laser length error covariance can be evaluated using $l = w_1 / w_2$:

$$\operatorname{var}[l] = \operatorname{cov}\left( \underbrace{\begin{bmatrix} \dfrac{1}{w_{20}} & -\dfrac{w_{10}}{w_{20}^{\ 2}} \end{bmatrix}}_{E_2} \begin{bmatrix} \Delta w_1 \\ \Delta w_2 \end{bmatrix} \right) = E_2 E_1 \operatorname{cov}[\eta_m] E_1^{\ T} E_2^{\ T}$$

The position determination error covariance is

$$\operatorname{cov}[\Delta^c p] = ({}^c u E_2 E_1) \operatorname{cov}[\eta_m]({}^c u E_2 E_1)^T$$

## IV. Alignment Errors

### A. Alignment Error Model

Translational and rotational co-alignment errors between the camera frame and laser frames are another error source. Their calibration requires adequate error models and these alignment error models are provided in this section. The misalignment of the laser frame relative to the camera frame can described by

$$\begin{cases} C_{lc} = (I - \tilde{\theta})\overline{C}_{lc} \\ {}^c p_{cl} = {}^c\overline{p}_{cl} + \delta \end{cases}$$

where ${}^c\overline{p}_{cl} = [x_{cl}, y_{cl}, z_{cl}]^T$ is the nominal (known) position vector of the laser frame origin and $\overline{C}_{lc}$ the nominal DCM from camera frame to laser frame, $\delta = [\delta_x, \delta_y, \delta_z]^T \in R^3$ and $\theta = [\theta_x, \theta_y, \theta_z]^T \in R^3$ the small translational and rotational alignment errors of the laser frame relative to the camera frame. In particular, the translational errors are expressed in the camera frame, but the rotational errors are given in the laser frame locally. The target location of the laser spot on the target surface is related to the misaligned laser LOS as follows:

$${}^c p = ({}^s\overline{p}_{cl} + \delta) + l\overline{C}_{cl}(I + \tilde{\theta})^l u$$

Expanding the equation above yields:

$${}^c p = \begin{bmatrix} x_{cl} + \delta_x + l\cos e_l \sin a_l - l\theta_x \sin a_l \sin e_l \\ y_{cl} + \delta_y - l\sin e_l - l\theta_x \cos e_l \\ z_{cl} + \delta_z - l\theta_y \sin a_l + l\cos a_l \cos e_l - l\theta_x \cos a_l \sin e_l \end{bmatrix}$$

### B. Truth Camera Measurements

Relating the truth position of the laser spot to the pixel measurements in the camera measurements, we have

$$c_m = \begin{bmatrix} h_{pix} \\ v_{pix} \end{bmatrix} = M \begin{bmatrix} {}^c p(1)/{}^c p(3) \\ {}^c p(2)/{}^c p(3) \\ 1 \end{bmatrix} = c(l, \delta, \theta)$$

Note that the measurement is independent of the rotation alignment error along the laser boresight direction ( $\theta_z$ ) as change in that direction does not change the laser spot location on the target surface. Define the calibration variable vector $q$ consisting of the three translational errors and two rotation errors. The Jacobian of the measurement equation, denoted as $H_0 = [\partial c / \partial q^T]$ , can be calculated and the measurement equation can be linearized as follows:

$$c_m = \begin{bmatrix} h_{pix} \\ v_{pix} \end{bmatrix} = c_{m0} + H_0 q + \eta_m + \mathrm{O}(q^2)$$

where $c_{m0}$ is the predicted nominal measurement without any alignment errors, and $\mathrm{O}(q^2)$ is the 2nd and higher-order error terms of the measurements.

## V. Alignment Error Estimation

If the distances of laser frame to the target is known via independent measurements, the camera image of the laser spot can be used to estimate the alignment errors. We shall provide two such estimation approaches in this section.

### A. Batch Estimation of Error Variables

For a calibration target with its distance to the laser frame, denoted as $\bar{l}_1$ , known, the predicted camera measurement $c_{m0}(\bar{l}_1)$ can be calculated. With the actual camera measurement $c_{m1}$, construct measurement error as

$$dc_1 \equiv c_{m1} - c_{m0}(\bar{l}_1) = H_0 q + \eta_m + \mathrm{O}(q^2)$$

If we repeat this for three different laser targets with three different measurements, we can stack the deltas as follows:

$$\begin{bmatrix} dc_1 \\ dc_2 \\ dc_3 \end{bmatrix} = \begin{bmatrix} c_{m1} - c_{m0}(\bar{l}_1) \\ c_{m2} - c_{m0}(\bar{l}_2) \\ c_{m3} - c_{m0}(\bar{l}_3) \end{bmatrix} = \underbrace{\begin{bmatrix} H_0(\bar{l}_1) \\ H_0(\bar{l}_2) \\ H_0(\bar{l}_3) \end{bmatrix}}_{H} q + \begin{bmatrix} \eta_{m1} \\ \eta_{m2} \\ \eta_{m3} \end{bmatrix} + \mathrm{O}(q^2)$$

where $H \in R^{6\times 5}$ . An estimate of the error variables can be obtained using the least-square method:

$$\hat{q} = (H^T H)^{-1} H^T \begin{bmatrix} dc_1 \\ dc_2 \\ dc_3 \end{bmatrix}$$

Note that the success of using the proposed approach to estimate the alignment variables relies on the observability of the measurements with different laser spot distances. The observability can be checked numerically with typical camera/laser design geometry and is illustrated in the example Section later in this paper.

### B. Newton-Raphson Approach

The error estimation problem can also be solved by an iterative process. Define the difference between the predicted and measured camera data as $d(l,q) \equiv c(l,q) - c_m$ . For three different measurements with the target at three distinctive locations $l_1, l_2, l_3$ , we can stack the 6 measurements to form

$$D(\hat{q}) \equiv \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} c(l_1,\hat{q}) - c_{m1} \\ c(l_2,\hat{q}) - c_{m2} \\ c(l_3,\hat{q}) - c_{m3} \end{bmatrix}$$

Using the Jacobian of the analytic function of the predicted camera measurements, we can use the Newton-Raphson approach to solve for the alignment error variables as follows:

$$\begin{cases} \hat{q}_0 = 0 \\ \hat{q}_1 = \hat{q}_0 - [J(\hat{q}_0)^T J(\hat{q}_0)]^{-1} J(\hat{q}_0)^T D(\hat{q}_0) \\ \vdots \\ \hat{q}_k = \hat{q}_{k-1} - [J(\hat{q}_{k-1})^T J(\hat{q}_{k-1})]^{-1} J(\hat{q}_{k-1})^T D(\hat{q}_{k-1}) \end{cases}$$

### *C. Sequential Minimum-Variance Estimator (Kalman Filter)*

The Newton-Raphson iterative approach does not take into consideration of the measurement noise's effect on the estimation accuracy. It is a batch estimate approach with predefined measurement sets and only use the numeric solution to iterate on the existing measurement sets.

The same estimation problem can also be formulated as a sequential estimation problem, using the last estimated error variables to correct the measurement matrix for the subsequent estimation. This approach utilizing an iterative estimator to take one measurement set at a time and the estimation takes the form of a minimum-variance estimator by optimally mixing the previous estimate with subsequent new measurements. Assume the alignment errors are unknown but deterministic. The estimation problem can be formulated as follows:

$$\begin{cases} \dot{q} = 0 \\ c_m = c(l,q) + \eta_m \end{cases}$$

The initial estimate of the alignment error covariance and the camera measurement error covariance can be estimated from the alignment tooling accuracy and camera specifications, and are denoted as

$$\begin{cases} \mathrm{cov}(\eta_m) = R \\ \mathrm{cov}(q - q_0) = P_0 \end{cases}$$

Further, assume the initial estimate of the alignment error is $\hat{q}_0 = 0$ . For a laser spot with its distance known, the knowledge of laser spot distance can be used to calculate

$$\begin{cases} \hat{c}_0 = \hat{c}(\bar{l}_1, \hat{q}_0) \\ \hat{H}_0 = H(\bar{l}_1, \hat{q}_0) \end{cases}$$

Construct the measurement difference:

$$\Delta(\bar{l}_1, \hat{q}_0) \equiv c_{m1} - \hat{c}_0 = c_{m1} - c(\bar{l}_1, \hat{q}_0)$$

The new estimate of the alignment errors can be optimally estimated as

$$\hat{q}_1 = \hat{q}_0 + K_1 \Delta(\bar{l}_1, \hat{q}_0)$$

where the update gain is computed as

$$K_1 = P_0 \hat{H}_0^{\,T} (\hat{H}_0 P_0 \hat{H}_0^{\,T} + R)^{-1}$$

The new estimated alignment error covariance is now

$$P_1 = (I - K_1 \hat{H}_0) P_0$$

Once the alignment errors are estimated using the first laser spot, the same process can be repeated for a second laser spot with its known distance $\bar{l}_2$. We can similarly calculate

$$\begin{cases} \hat{c}_1 = \hat{c}(\bar{l}_2, \hat{q}_1) \\ \hat{H}_1 = H(\bar{l}_2, \hat{q}_1) \end{cases}$$

The same update procedure can be applied to any new camera measurements and the alignment estimation. Eventually, the final estimate of the alignment errors can be obtained when enough measurements are fed into this process. In addition, the propagated estimation error covariance matrix gives an estimate of the alignment error post-calibration residual magnitude and, most importantly, that information can be used to predict the target position determination accuracy.

## VI. TARGET POSITION DETERMINATION CALIBRATION

The purpose of camera-laser alignment error estimation is to improve target positon determination performance. Once the knowledge of the alignment errors is improved, the new information can be used to refine the position determination accuracy. This section provides details on how position calibration can be accomplished.

### *A. Lookup Table vs. Intrinsic Error Parameters*

There are two fundamentally different approach for improving position determination accuracy. Perceivably, a three component position correction delta can be obtained *a priori* for all possible target locations within the required field-of-regard and depth coverage of the PIXL instrument. This approach can use a lookup table to store all possible correction sets and at the instrument operational time the position correction can be performed based on the knowledge of the actual pose. This lookup table requires a tremendous amount the storage, possibly on the mission computer, and suffers from quantization errors if there are large gaps among the discrete table entries.

A different approach makes use of the estimated intrinsic values of the alignment errors and calculate the required position corrections in real-time via simple algebraic equations. This approach is described in this section.

### *B. Position Determination with Alignment Errors*

In previous sections, target position for nominal cases (without alignment errors) is determined by solving the equations for $w_1, w_2$ . With alignment errors in this system, the position determination algorithm needs to be slightly modified. Consider the following equation that relates the camera measurements with the estimated alignment error variables at PIXL operational time:

$$c_m = \frac{\begin{bmatrix} h_{pix0} - f(x_{cl} + \hat{\delta}_x + l(\cos e \sin a - \hat{\theta}_x \sin a \sin e + \hat{\theta}_y \cos a)) \\ v_{pix0} - f(y_{cl} + \hat{\delta}_y - l(\sin e + \hat{\theta}_x \cos e)) \end{bmatrix}}{s(z_{cl} + \hat{\delta}_z + l(\cos a \cos e - \hat{\theta}_x \cos a \sin e - \hat{\theta}_y \sin a))}$$

Target position determination becomes trivial calculation if the laser distance can be determined from the camera measurements from the above equation. Note that the above equation consists of two separate equations with a single unknown variable $l$. We can proceed to easily derive the two solutions for $l$ from the horizontal and vertical measurements of the camera. Once the laser distance is estimated, it is straightforward to calculate the position as

$$\begin{cases} \hat{p}_h = \bar{p}(\hat{q}) + \hat{l}_h \bar{u}(\hat{q}) \\ \hat{p}_v = \bar{p}(\hat{q}) + \hat{l}_v \bar{u}(\hat{q}) \end{cases}$$

The detailed version of these equations are omitted in this paper due to space limitation.

### C. Position Determination Errors and Sensitivity

In general, the positions computed from the two equations should only differ with uncertainties associated with measurement errors and the alignment calibration residuals. A natural question to ask is which solution gives a better solution in terms of the determination accuracy. To answer this question, it is required to evaluate the position determination error metric as a function of the input errors for the two solutions. The input errors in the determination of the position during instrument operational time include the real-time measurement errors and the alignment calibration residuals.

The covariance of the position determination errors can be evaluated as follows. Define the computed position sensitivity matrix to the measurement as

$$\begin{cases} J_{p_h m} \equiv \dfrac{\partial \hat{p}_h}{\partial c_m^{\ T}} = \begin{bmatrix} j_{p_h m}(1{,}1) & 0 \\ j_{p_h m}(2{,}1) & 0 \\ j_{p_h m}(3{,}1) & 0 \end{bmatrix} \in R^{3\times 2} \\ J_{p_v m} \equiv \dfrac{\partial \hat{p}_v}{\partial c_m^{\ T}} = \begin{bmatrix} 0 & j_{p_v m}(1{,}2) \\ 0 & j_{p_v m}(2{,}2) \\ 0 & j_{p_v m}(3{,}2) \end{bmatrix} \in R^{3\times 2} \end{cases}$$

The sensitivity to the alignment calibration residuals is

$$\begin{cases} J_{p_h q} \equiv \dfrac{\partial \hat{p}_h}{\partial q^T} \in R^{3\times 5} \\ J_{p_v q} \equiv \dfrac{\partial \hat{p}_v}{\partial q^T} \in R^{3\times 5} \end{cases}$$

The covariance of the position determination error can be computed approximately as

$$\begin{cases} \mathrm{cov}(\delta\hat{p}_h) \approx J_{p_h m} \mathrm{cov}(\eta_m) J_{p_h m}^{\ T} + J_{p_h q} \mathrm{cov}(\delta\hat{q}) J_{p_h q}^{\ T} \\ \mathrm{cov}(\delta\hat{p}_v) \approx J_{p_v m} \mathrm{cov}(\eta_m) J_{p_v m}^{\ T} + J_{p_v q} \mathrm{cov}(\delta\hat{q}) J_{p_v q}^{\ T} \end{cases}$$

where $\mathrm{cov}(\delta\hat{q}) = P$ is the alignment calibration residual covariance matrix and $\mathrm{cov}(\eta_m) = R$ is the measurement noise covariance matrix.

### D. Optimal Combination of Two Position Estimates

For the two estimates of the position, the best linear unbiased estimator of the position is a linear combination of the two estimates, i.e.

$$\hat{p} = b_h \hat{p}_h + b_v \hat{p}_v$$

where

$$\begin{cases} b_h = \dfrac{tr(J_v C J_v^{\ T})}{tr(J_h C J_h^{\ T}) + tr(J_v C J_v^{\ T})} \\ b_v = \dfrac{tr(J_h C J_h^{\ T})}{tr(J_h C J_h^{\ T}) + tr(J_v C J_v^{\ T})} \end{cases}$$

with

$$\begin{cases} J_h \equiv \begin{bmatrix} J_{p_h m} & J_{p_h q} \end{bmatrix} \\ J_v \equiv \begin{bmatrix} J_{p_v m} & J_{p_v q} \end{bmatrix} \end{cases} \text{and } C \equiv \mathrm{cov}\begin{bmatrix} \eta_m \\ \delta\hat{q} \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & P \end{bmatrix}$$

## VII. CALIBRATION WITH KNOWLEDGE OF PXIL MOTION

### A. PIXL Motion Control for Subsequent Distance Knowledge

So far, we have been assuming the knowledge of the laser distance is provided by an external independent measurement device. An interesting question is, instead of using target location knowledge provided by external measurements, can we use PIXL motion control capability with its kinematic knowledge to provide the knowledge of the laser spot distance? It is noted that as both camera and laser are fixed to the PIXL sensor head, their co-alignment is independent of any sensor head motion.

With this observation, it is perceivable that we can move the PIXL using its steering/control facility with a known displacement from the original PXIL/target configuration. In this case, proposed approach needs only an one-time measurement of the laser spot distance. The subsequent laser distance can be obtained from the PIXL motion commanding facility to facilitate a similar calibration process. It is noted, however, the mechanical errors of the PIXL system (strut lengths, joint locations, etc.) can impact the accuracy of the PIXL commanded displacement, and therefore affects the knowledge in the predicted camera measurement variable $\hat{c}$.

### B. Motion Uncertainty's Impact on Calibration

This PIXL motion errors can be evaluated using the PIXL pose accuracy analysis. The analysis can be performed using the results of the PIXL calibration procedure. The PIXL calibration is a separate topic from this paper and its approach has been proposed elsewhere [6]. If PIXL motion predictions are used in lieu of external measurements to support the camera/laser calibration addressed in this paper, its error can be incorporated to modify the measurement equations and treated as part of the measurement errors:

$$\hat{c} = \bar{\hat{c}} + \eta_{PIXL}$$

$$\Delta(\bar{l}_1, \hat{q}_0) \equiv c_{m1} - \bar{\hat{c}}_0 = H(\bar{l}_1, \hat{q}_0)q + \eta_m = \hat{H}_0 q + \eta_m + \eta_{PIXL}$$

It is noted, however, while the camera measurement error $\eta_m$ are most likely to be noise, the PIXL motion knowledge error is more to be pose-dependent unknown biases and this error characteristics may invalidate measurement error assumption for the minimum-variance estimation. Geometric diversity of poses for laser spot measurement may mitigate some of the problems introduced by the PIXL motion error.

## VIII. A NUMERICAL EXAMPLE

### A. *Assumed PIXL Camera/Laser Geometry Parameters*

The proposed algorithms are evaluated in this section to show the alignment error calibration performance and the position determination accuracy. The following geometry parameters are assumed for a camera and a single laser configuration.

| | | |
|---|---|---|
| f=12 | (mm) | camera focus length |
| e=-5 | (deg) | nominal elevation angle of laser frame |
| a=30 | (deg) | nominal azimuth angle of laser frame |
| x0=-43 | (mm) | nominal laser frame origin x-coordinate |
| y0=5 | (mm) | nominal laser frame origin x-coordinate |
| z0=0 | (mm) | nominal laser frame origin x-coordinate |

### B. *Error Models*

A set of the randomly chosen alignment errors are used to create the truth model of the SLS. The estimated alignment errors are then compared to the chosen truth to see the effect of estimation. Position determination results are then provided based on the assumed measurement noise and post-calibration residual. For simulation purpose, the following error parameters are used:

| | | |
|---|---|---|
| Tr_sigma = 1 | (mm) | translational alignment error sigma |
| Ro_sigma = 0.5 | (deg) | rotational alignment error sigma |
| Nh_sigma = 0.007 | (mm) | horizontal camera centroid noise |
| Nv_sigma = 0.007 | (mm) | vertical camera centroid noise |

Two approaches are used to show the alignment error estimation. Those results are presented next.

### C. *Batch Estimation*

The first approach uses batch estimation with Newton-Raphson iterations. Without measurement noise, the batch estimation can be shown to have very satisfactory performance. However, if noise is introduced, the estimation results are less then perfect and show some residual biases which can be due to the limited number of samples (4 cases in our example) and to some extend the limitation of the approach itself.

### D. *Sequential Estimator*

The second approach uses the sequential minimum-variance estimator with the measurement noise incorporated in the emulation of the measurements. The results of the estimation convergence are shown as below.

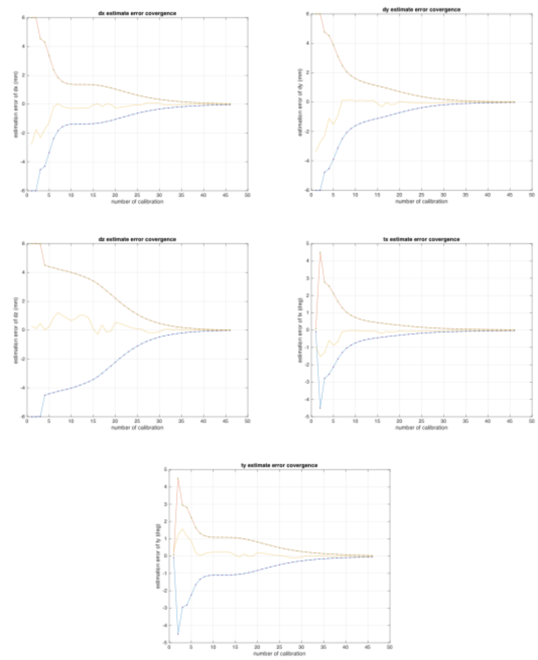

Figure 2 – Alignment Error Estimate Convergence

## IX. CONCLUSION

This paper describes several calibration approaches for the alignment errors between the camera and lasers of the PIXL instrument sensor head for NASA JPL's Mar2020 Rover Project. Future work will include using the approach with test data obtained from laser trackers and CMM machine and developing PIXL hexapod calibration techniques.


## ACKNOWLEDGMENTS

The author would like to thank NASA JPL's Mars 2020 PIXL Instrument Project for the support.


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